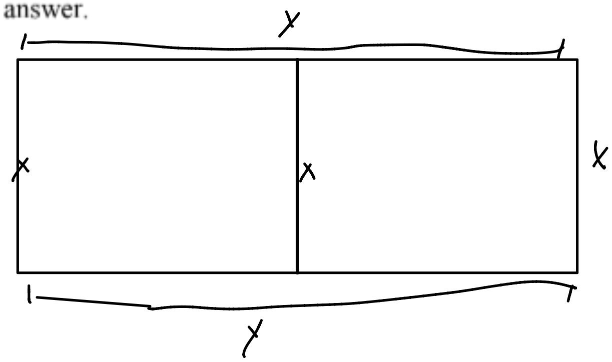
Period 3, Jan 15, 2025

3. A farmer has 360 feet of fencing to enclose two adjacent and identical rectangular corrals. What dimensions should be used to maximize the enclosed area? What is the maximum area? Include units in your answer.



$3x + 2y = 360$

$A = x \cdot y$

$3x = 360 - 2y$

$x = 120 - \frac{2}{3}y$

$A = (120 - \frac{2}{3}y)y$

$A = 120y - \frac{2}{3}y^2$

$0 = 120 - \frac{4}{3}y$

$\frac{4}{3}y = 120$

Keep going

8. Given $g(x) = ln(x + 1)$, use the equation of the tangent line of $g(x)$ at $x = 0$ to find the approximation of $ln(1.1)$.

$g(0) = \ln(0+1) = \ln(1) = 0$

Point $(0, 0)$

$g'(x) = \frac{1}{x+1} \cdot 1 = \frac{1}{x+1}$

$g'(0) = \frac{1}{0+1} = \frac{1}{1} = 1 =$ slope

$y = mx + b$

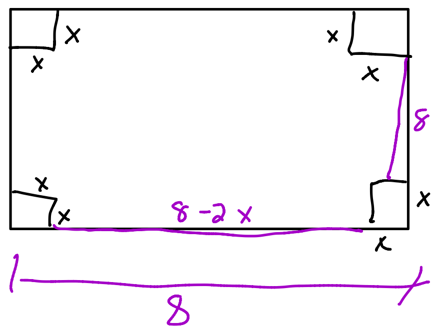
$0 = 1 \cdot 0 + b$

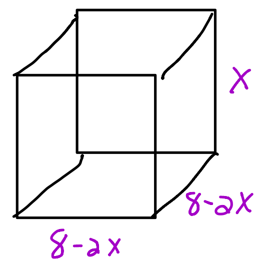
$b = 0$

$y = x$

$g(0.1) = \ln(1.1) \approx 0.1$

5. A tinsmith wishes to make an open top box from a square piece of tin which measures 8 in. by 8 in. To accomplish this task, he proposes to cut equal square pieces from each corner of the tin and fold up the tin to form sides. Determine the length of the sides of the squares to be cut from the corners so that the box will have the greatest possible volume.





$V = (8-2x)(8-2x)\cdot x = (64 - 32x + 4x^2)x$

$V = 4x^3 - 32x^2 + 64x$

$\frac{dV}{dx} = 12x^2 - 64x + 64$

$4(\underbrace{3x^2 - 16x + 16}_{3\cdot 16 = 48}) = 0$

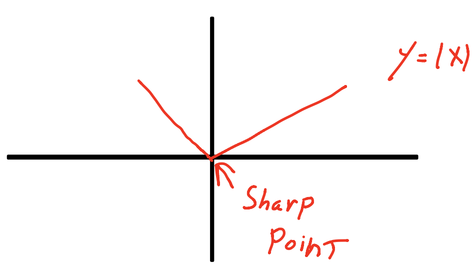
$-12 \quad -4$

$3x^2 - 12x - 4x + 16$

$3x(x-4) - 4(x-4)$

$4(x-4)(3x-4) = \frac{dV}{dx} = 0$

6. Determine if the Mean Value Theorem applies to the function $f(x) = |x|$ over the interval $[-2, 2]$. If it does not, explain why. If it does, find the value of c guaranteed by the theorem.



2. Find two positive numbers such that their product is 192 and the sum of the first plus three times the second is a minimum.

a, b

$a \cdot b = 192$

$a = \frac{192}{b}$

Find a

$a + 3b = m$

$\frac{192}{b} + 3b = m$

$192 \cdot b^{-1} + 3b = m$

$-192b^{-2} + 3 = 0$

$b^2 \cdot 3 = \frac{192}{\cancel{b^2}} \cdot \cancel{b^2}$

$\frac{\cancel{3}b^2}{\cancel{3}} = \frac{192}{3}$

$b^2 = 64$

$b = \pm 8$

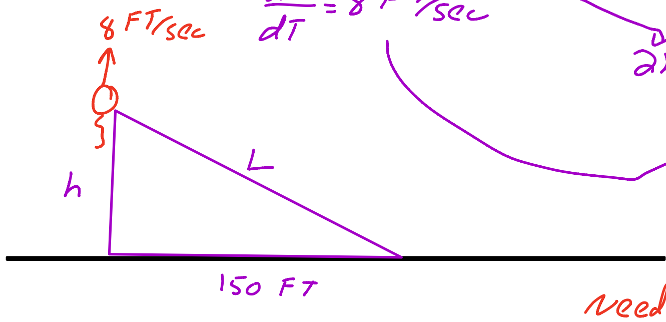
$b = 8$

8. Given $g(x) = ln(x + 1)$, use the equation of the tangent line of $g(x)$ at $x = 0$ to find the approximation of $ln(1.1)$.

7. A small balloon is released at a point 150 feet away from an observer who is on level ground. If the balloon goes straight up at the rate of 8 ft/sec, how fast is the distance from the observer to the balloon increasing when the balloon is 50 ft. high?

$h=50$

$\frac{dh}{dT} = 8 \text{ FT/sec}$



$h^2 + 150^2 = L^2$

$2h\frac{dh}{dT} + 0 = 2L\frac{dL}{dT}$

$\frac{dL}{dT} = ?$

need L value when h=50

$50^2 + 150^2 = L^2$

$2500 + 22500 = L^2$

$25000 = L^2$

$\sqrt{25000} = L$

$\sqrt{2500 \cdot 10} = L$

$50\sqrt{10} = L$

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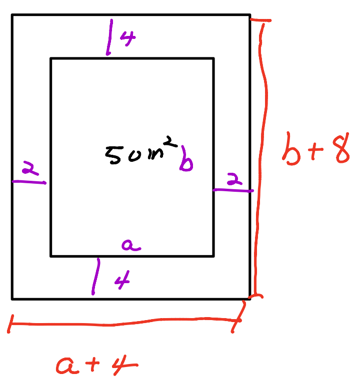
4. A poster is to contain 50 square inches of printed matter with margins of 4 inches at both the top and the bottom and 2 inches at each side. Find the dimensions that will minimize the total area of the poster.



$ab = 50$

$a = \frac{50}{b}$

$A = (b+8)(a+4)$

$A = (b+8)\left(\frac{50}{b}+4\right)$

$A = 50 + 4b + \frac{400}{b} + 32 = 82 + 4b + 400b^{-1}$

$\frac{dA}{db} = 0 = 0 + 4 - 400b^{-2} \Rightarrow \frac{400}{b^2} = 4$

$b = \pm 10$

$$y = 4x^6 + 7x^2 - 9x + 12$$

$$\frac{dy}{dx} = 4 \cdot 6x^{6-1} + 7 \cdot 2x^{2-1} - 9 \cdot 1x^{1-1} + 0$$

$$\cancel{dx} \cdot \frac{dy}{\cancel{dx}} = (24x^5 + 14x - 9) \cdot dx$$

$$\int dy = \int (24x^5 + 14x - 9)\,dx$$

$$y = 24 \cdot \frac{1}{6} \cdot x^{5+1=6} + 14 \cdot \frac{1}{2} \cdot x^{1+1=2} - 9 \cdot \frac{1}{1} \cdot x^{0+1=1} + C$$

$$y = 4x^6 + 7x^2 - 9x + C$$

$$\frac{d}{d\theta}[\sin\theta] = \cos\theta \qquad \int \cos\theta \, d\theta = \sin\theta + C$$

$$\int \left(\frac{3}{2}x^4 + 6x^5 - 12x\right)dx$$

$$\frac{3}{2} \cdot \frac{1}{5} \cdot x^{4+1=5} + \cancel{6} \cdot \frac{1}{\cancel{6}} \cdot x^{5+1=6} - 12 \cdot \frac{1}{2} \cdot x^{1+1=2} + C$$

$$\frac{3}{10}x^5 + x^6 - 6x^2 + C$$

$$\int 7\,dx = 7x + C$$

$$\int\left(\frac{7x^4+12x^2-15x+60x^5}{x^2}\right)dx$$

$\int \frac{1}{x}dx = \ln|x|$

$$\int\left(\frac{7x^4}{x^2}+\frac{12x^2}{x^2}-\frac{15x}{x^2}+\frac{60x^5}{x^2}\right)dx$$

$$\int\left(7x^2+12-\frac{15}{x}+60x^3\right)dx$$

$$7\cdot\frac{1}{3}\cdot x^{2-1=3}+12x-15\cdot\ln|x|+60\cdot\frac{1}{4}\cdot x^{3+1=4}+C$$

$$\frac{7}{3}x^3+12x-15\ln|x|+15x^4+C$$

$$\int(\sin x+\sec^2 x)\,dx=\int\sin x\,dx+\int\sec^2 x\,dx=-\cos x+\tan x+C$$

$$\int\left(\sqrt[3]{x^4}+x^2-\frac{4}{x}\right)dx=\int\left(x^{\frac{4}{3}}+x^2-\frac{4}{x}\right)dx$$

$$\cdot\frac{3}{7}\cdot x^{\frac{4}{3}+\frac{3}{3}=\frac{7}{3}}+\cdot\frac{1}{3}\cdot x^{2+1=3}-4\ln|x|+C$$

$F(x) = 4x^3 - 7x^2 + 2x - 5$

when $x=2$ $\quad y=3$

$F(2) = 32 - 28 + 4 - 5 = 3$

$F'(x) = 12x^2 - 14x + 2$

$\int (12x^2 - 14x + 2)\,dx = 4x^2 - 7x + 2x + C$

when $x=2 \Rightarrow y=3$

$y = 4x^3 - 7x^2 + 2x + C$

$3 = 4(2)^3 - 7(2)^2 + 2(2) + C$

$3 = 32 - 28 + 4 + C$

$3 = 8 + C$

$-5 = C$

$\dfrac{d^2y}{dx^2} = 4x^2 + 2x - 6 = F''(x)$

$F'(3) = 9$

$F(1) = 12$

Find $F(x)$

$F'(x) = \int (4x^2 + 2x - 6)\,dx$

$F'(x) = 4 \cdot \frac{1}{3} \cdot x^{2+1=3} + 2 \cdot \frac{1}{2} \cdot x^{1+1=2} - 6x + C_1$

$F(x) = \int F'(x)\,dx$

$F'(x) = \frac{4}{3}x^3 + x^2 - 6x + C_1$

$F(x) = \int \left(\frac{4}{3}x^3 + x^2 - 6x - 18\right)dx$

$9 = \frac{4}{3}(3)^3 + 3^2 - 6(3) + C_1$

$F(x) = \frac{1}{3}x^4 + \frac{1}{3}x^3 - 3x^2 - 18x + C_2$

$9 = 36 + 9 - 18 + C_1$

$-18 = C_1$

$12 = \frac{1}{3}(1)^4 + \frac{1}{3}(1)^3 - 3(1)^2 - 18(1) + C_2$

$12 = \frac{2}{3} - 21 + C_2 \Rightarrow C_2 = 32\frac{1}{3}$

$F(x) = \frac{1}{3}x^4 - \frac{1}{3}x^3 - 3x^2 - 18x + 32\frac{1}{3}$

$S(T) = \text{Position}$

$S'(T) = V(T) = \text{Velocity}$

$S''(T) = V'(T) = a(T) = \text{accel}$

$\int V(T)\,dT = S(T)$

$\int a(T) = V(T)$

Form A

$\frac{dx}{dT} = 3 \text{ unit}/\text{min}$

Form B

$\frac{dx}{dT} = 4 \text{ unit}/\text{min}$

①

$y = \sqrt[3]{x^2+1}$

$y = (x^2+1)^{\frac{1}{3}}$

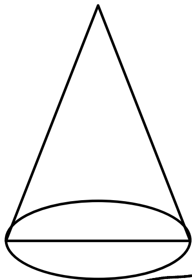
$\frac{dy}{dT} = \frac{1}{3}(x^2+1)^{\frac{1}{3}-1} \cdot 2x \cdot \frac{dx}{dT}$

$\frac{dy}{dT} = \frac{2x}{3\sqrt[3]{(x^2+1)^2}} \frac{dx}{dT}$

Find $\frac{dy}{dT}$ when $(x=0, y=1)$

$\frac{2 \cdot 0}{3\sqrt[3]{(0^2+1)^2}} \cdot 3 = 0 = \frac{dy}{dT}$

(2)



Form A

$\frac{dV}{dT} = 12 \ FT^3/min$

Form B

$\frac{dV}{dT} = 11 \ FT^3/min$

$\boxed{r = \frac{1}{3} \cdot h}$

Find $\frac{dh}{dT}$ when $h = 5$

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi\left(\frac{h}{3}\right)^2 \cdot h = \frac{\pi}{27}h^3$

$\frac{dV}{dT} = \frac{\pi}{27} \cdot 3h^2 \frac{dh}{dT} = \frac{\pi}{27} \cdot 3 \cdot 5^2 \cdot \frac{dh}{dT} = \frac{25\pi}{9}\frac{dh}{dT}$

Form A

$\frac{dV}{dT} = 12 \ FT^3/min$

$\frac{9}{25\pi} \cdot 12 = \cancel{\frac{25\pi}{9}}\left(\frac{dh}{dT}\right) \cdot \cancel{\frac{9}{25\pi}}$

$\frac{108}{25\pi} \ FT/min = \frac{dh}{dT}$

Form B

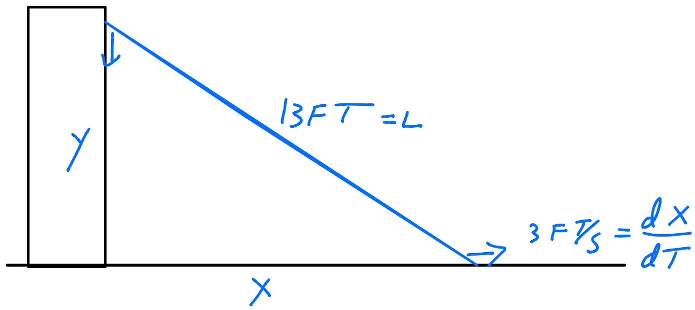
$\frac{dV}{dT} = 11 \ FT^3/min$

$\frac{9}{25\pi} \cdot 11 = \cancel{\frac{25\pi}{9}} \cdot \frac{dh}{dT} \cdot \cancel{\frac{9}{25\pi}}$

$\frac{99}{25\pi} \ FT/min = \frac{dh}{dT}$

3)

$\frac{dL}{dT} = 0$



If $Y = 12$ what is X?

$X^2 + Y^2 = 13^2$

$2x\frac{dx}{dt} + 2y\frac{dy}{dT} = 0$

$2\cdot5\cdot3 + 2\cdot12\cdot\frac{dy}{dT} = 0$

$24\frac{dy}{dT} = -30$

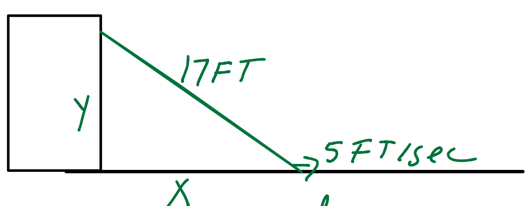
$\frac{dy}{dT} = \frac{-30}{24}$ FT/S $= \frac{-5}{4}$ FT/S

$12^2 + X^2 = 13^2$

$144 + X^2 = 169$

$X^2 = 25$

$X = 5$



$\frac{dx}{dT} = 5$ FT/S

$Y = 15$

$X = ? = 8$

$X^2 + 15^2 = 17^2$

$X^2 + 225 = 289$

$X^2 = 64$

$X = 8$

$X^2 + Y^2 = 17^2$

$2x\frac{dx}{dT} + 2y\frac{dy}{dT} = 0$

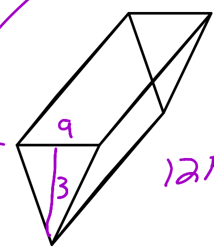
$2\cdot8\cdot5 + 2\cdot15\cdot\frac{dy}{dT} = 0$

$80 + 30\frac{dy}{dT} = 0$

$30\frac{dy}{dT} = -80$

$\frac{dy}{dT} = -\frac{8}{3}$ FT/sec

Form A



$b=3h$

$\frac{dh}{dT} = \frac{1}{4}$ in/min when $h=2$

$12FT = 144in$

$V = \frac{1}{2} \cdot b \cdot h \cdot L$

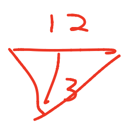
$V = \frac{1}{2} \cdot 3h \cdot h \cdot 144$

$V = 216h^2$

$\frac{dV}{dT} = 432h\frac{dh}{dT} \Rightarrow \frac{dV}{dT} = 432 \cdot 2 \cdot \frac{1}{4}$

216

Form b



$b = 4 \cdot h$

$V = \frac{1}{2} \cdot b \cdot h \cdot 144$

$V = \frac{1}{2} \cdot 4h \cdot h \cdot 144$

$V = 288h^2$

$\frac{dV}{dT} = 576h\frac{dh}{dT} = 576 \cdot 2 \cdot \frac{1}{4}$

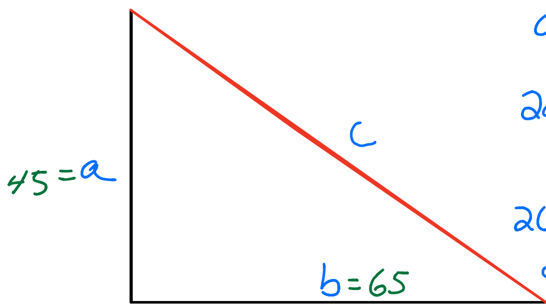
288

Form A

After 1 hour

north

$45\ \text{mph} = \frac{da}{dT}$



65 mph East

$\frac{db}{dT} = 65$

$a^2 + b^2 = c^2$

$2a\frac{da}{dT} + 2b\frac{db}{dT} = 2c\frac{dc}{dT}$

$2(45)45 + 2\cdot 65\cdot 65 = 2\cdot 25\sqrt{10}\cdot\frac{dc}{dT}$

$4050 + 8450 = 50\sqrt{10}\,\frac{dc}{dT}$

$\frac{12500}{50\sqrt{10}} = \frac{\cancel{50\sqrt{10}}\,\frac{dc}{dT}}{\cancel{50\sqrt{10}}}$

$\frac{250}{\sqrt{10}} = \frac{dc}{dT}$

$\frac{250}{\sqrt{10}}\ \text{mph} = \frac{dc}{dT}$

$45^2 + 65^2 = c^2$

$2025 + 4225 = c^2$

$6250 = c^2$

$25\sqrt{10} = c$

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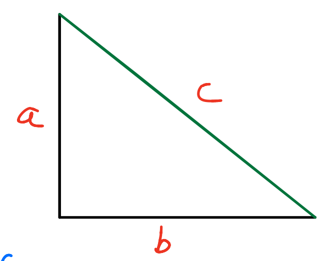
5 Form B

$\frac{da}{dT} = 45\ \text{mph}$



after 1 hour

a = 45 miles

b = 70 miles

$\frac{db}{dT} = 70\ \text{mph}$

$a^2 + b^2 = c^2$

$\frac{2a\frac{da}{dT} + 2b\frac{db}{dT}}{2} = \frac{2c\frac{dc}{dT}}{2}$

$a\cdot 45 + b\cdot 70 = c\frac{dc}{dT}$

$45\cdot 45 + 70\cdot 70 = 5\sqrt{277}\,\frac{dc}{dT}$

$\frac{6925}{5\sqrt{277}} = \frac{\cancel{5\sqrt{277}}\,\frac{dc}{dT}}{\cancel{5\sqrt{277}}}$

$\frac{1385}{\sqrt{277}}\ \text{mph} = \frac{dc}{dT}$

$45^2 + 70^2 = c^2$

$2025 + 4900 = c^2$

$6925 = c^2$

$\to 5\sqrt{277} = c$